

## **5 p-channel Devices**

### **5.1 Abstract**

It is desirable, as explained in Chapter 2, to increase the performance of holes to match electrons at room temperature within a semiconductor system, and strained silicon-germanium heterostructures are promising in this regard. However, this promise is not always fulfilled. It would be useful to be able to find out how to optimize the design and growth of such structures rather than rely on trial and error, and low-temperature characterization of the p-type silicon-germanium system is investigated with this in mind. Even though the results in terms of fundamental 2-dimensional carrier gas physics may not seem directly applicable to room-temperature electronics (despite being interesting in their own right) information about scattering mechanisms and localization may lead to higher quality material for commercial device production.

### **5.2 Introduction**

Two contrasting p-channel devices are investigated in this chapter: the first is a research-scale device grown by MBE with a  $\text{Si}_{0.8}\text{Ge}_{0.2}$  active channel and inverted doping; the second is a commercial-scale device grown by CVD with a  $\text{Si}_{0.5}\text{Ge}_{0.5}$  active channel and n-type doping throughout. Both have gates so that the sheet carrier concentration can be modulated, and are in standard Hall-bar geometries.

### **5.3 Structure of Devices**

#### **5.3.1 Research Scale Device on Wafer 55/53**

This device is a simple heterostructure with doping below the alloy and a metal gate. In contrast to the Siemens device (below) its large size makes it relatively uncomplicated to handle and measure. The structure of this device is shown in Figure 5.1.

Figure 5.1 The structure of sample 55/53, a research scale p-channel device featuring inverted doping and a pseudomorphic alloy layer.

This device is a heterostructure with the same alloy composition as the coupled-channel devices that will be considered in Chapter 7. The buffer layer was grown at 700°C, the dopant layer and the spacer were grown at 575°C, and the alloy and cap layers were grown at 610°C. Contacts were formed by Al sputtering and annealing at 470-520°C in a nitrogen atmosphere. There is a Ti/Al sputtered Schottky gate contact: the device is an inverted structure (there is modulation doping below the active channel) so the sheet density in the channel can be varied with gate bias.<sup>1</sup>

### 5.3.2 Commercial Scale Siemens Device

The common length scale of devices created for research is of the order of at least a few hundred microns (as with 55/53, and the devices investigated in following chapters) but devices fabricated in commercial integrated circuits are at most a few microns in size (depending on the application)<sup>2,3</sup> so it is important to investigate such small devices in a research context. Also, this second device was grown with a commercial reduced-pressure CVD process, not a research-laboratory MBE system.

The device is very sensitive, due to its small size, both in terms of the geometry of the Hall bar and the thickness of the heterostructure layers: there is a high degree of strain associated with a Si<sub>0.5</sub>Ge<sub>0.5</sub> layer grown pseudomorphically on silicon which limits the thickness of the alloy layer.

The structure of this device is shown in Figure 5.2. The samples used to obtain the results presented here were grown using CVD, by Siemens, and took the form of gated Hall bars. A 5nm layer of Si<sub>0.5</sub>Ge<sub>0.5</sub> was grown pseudomorphically on Si, capped by 5nm of Si. Whilst this alloy thickness exceeds the Matthews-Blakeslee equilibrium critical thickness of ~4nm,<sup>4,5,6</sup> and the presence of the cap stabilizes the epitaxial alloy only slightly against strain relaxation,<sup>7</sup> accounting for interactions between dislocations more correctly gives an equilibrium critical thickness of 8nm.<sup>8</sup>

Figure 5.2 The structure of the Siemens device.

An n-type (phosphorus) dopant level of  $2 \times 10^{17} \text{cm}^{-3}$  is present throughout: the structure is not modulation doped but the donor concentration may be higher than this deep below the active channel. The gate oxide is 3.8nm thick. The Hall bars were created on a commercial device scale; they are  $2.5 \mu\text{m}$  wide and  $12.4 \mu\text{m}$  long overall, with the terminals spaced  $6.6 \mu\text{m}$  apart symmetrically. This means that there is no appreciable correction to the Hall potential due to the device geometry.<sup>9</sup>

## 5.4 Experimental Considerations

### 5.4.1 Current Heating

Measurements of the Hall effect at currents of 20nA, 50nA and 100nA (at 350mK) were performed on device 55/53 (see Figures 5.8 and 5.9) and the results are discussed in section 5.4.2. IV measurements showed no change in resistance of the device as a function of current, up to at least 100nA, implying that there was no heating effect from the current.

During measurements on the Siemens device, a current of 10nA was generally used, to avoid heating the carriers above the lattice temperature. The small size of the device implies a high current density. The energy loss rate per carrier was calculated to be of the order of  $10^{-17} \text{W}$ , but this should be low enough that thermal carrier diffusion to the contact regions equalises the carrier and lattice temperatures.<sup>10</sup>

### 5.4.2 ESD Protection

The breakdown field of silicon dioxide is of the order of  $10^8 \text{Vm}^{-1}$ .<sup>11</sup> This means that the gate oxide of the Siemens devices, which is a few nanometres thick, is in danger of being compromised when subject to potential differences of just a few tens of volts. Not only must great care be taken to protect the devices from electrostatic discharge (by storing the devices with the pins of the 8-pin DIL packages embedded in conducting foam or aluminium foil-wrapped polystyrene, for example, as is common practice with CMOS integrated circuits) but experimental procedures must be designed

so as to keep voltages at safe levels at all times.

It is not sufficient merely to short-circuit the gate and a single other contact: since the devices operate in enhancement mode, when  $V_{GS}$  is zero all the Hall terminals are electrically isolated from each other, and a large enough potential between any one Hall terminal and the gate will destroy the device. Protection was also considered necessary during the measurement procedure, since otherwise devices were surviving no more than a few days.

The protection circuit shown in Figure 5.3 was built and incorporated into the junction box between the cryostat system and the measurement electronics. The gate was protected with a pair of Zener diodes (each rated at 2.7V wired back-to-back) and a low-pass filter circuit with a cut-off frequency of around 200Hz; each of the six Hall terminals was protected by a pair of silicon diodes wired in opposed parallel. The IV characteristics of the gate circuit are shown in Figure 5.4, and the IV characteristics of a sample contact circuit are shown in Figure 5.5. For voltage levels considered reasonable in terms of normal device operation and measurement (a few millivolts for each Hall terminal, and up to 2.5V on the gate) the protection circuitry should have no bearing on the results. The gate protection circuitry does, however, prevent measurement of gate leakage current.

Additionally, all terminals were short-circuited to ground between measurements.

## 5.5 Results from 55/53

As this device uses a Schottky gate contact, its operation relies on the silicon cap behaving as a non-conducting dielectric. However, at temperatures above around 60K current can flow more or less freely between the gate and the active channel restricting the useful measurement range to below this temperature. At temperatures below 60K the gate system behaves as a Schottky diode so the range of gate voltages is limited. The 350mK characteristics of the gate are shown in Figure 5.6.

Figure 5.3 The circuit diagram of the ESD protection system: the gate is protected from short spikes and sudden changes by a low-pass filter (with a time constant of the order of a few milliseconds) whilst the back-to-back Zener diode arrangement ensures that  $V_{GS}$  is limited to a safe value. The two current and four voltage contacts are each protected by a pair of silicon diodes in anti-parallel which prevent potentials of more than a few hundred millivolts from building up.

Figure 5.4 The direct-current transfer characteristic of the gate protection circuit in Figure 5.3. It can be seen that as increasing gate voltages are applied, current (dashed line) begins to flow through to earth through the Zener diodes, and the voltage presented to the gate of the device (solid line) is reduced. For small applied voltages, the leakage current is negligible.

Figure 5.5 The direct-current transfer characteristic of the contact protection circuit in Figure 5.3. As the applied voltage reaches a few hundred millivolts, the current (dashed line) through the silicon diodes quickly reaches the 1mA compliance limit of the HP4148 performing the measurement and the voltage presented to the device (solid line) saturates. For small applied voltages, the leakage current is negligible.

Figure 5.6 The IV characteristics of the gate of the device on wafer 55/53 at 350mK, showing leakage similar to that shown by a Schottky diode. At 2.7V, the current reaches the set compliance limit of the HP parameter analyzer. Operation was limited to gate voltages in the range -1V to +1V.

On the basis of this, gate voltages were kept within the range -1V to +1V. The relationship between gate voltage and sheet density was roughly linear, as will be seen.

### 5.5.1 Resistivity as a Function of Temperature

A summary of the  $\rho(T)$  characteristics of this device (in the absence of magnetic field) is shown in Figure 5.7. For larger, “metallic” sheet densities,  $\rho(T)$  weakly saturates as the temperature goes to zero, but features a local maximum at  $T_F/3$ , where  $T_F$  is the Fermi temperature as defined in Equation 3.5. However, as will be discussed in terms of similar data from the Siemens device, current theories of the behaviour of  $\rho(T)$  are based on screening and interactions which should lead to a minimum in resistance at a temperature of the order of 1K, and a divergence at zero temperature.<sup>12</sup>

### 5.5.2 Mobility

Hall measurements were performed, and the single-carrier analysis described in Chapter 4 was applied. There should be no complications from multiple carrier gases since, for the device's gate to operate, the unstrained silicon layers must be frozen out and this includes the doped layer. The only free carriers available for conduction should be those in the 2DHG. Results for the sheet density as a function of gate voltage at 350mK are shown in Figure 5.8. A field of 0.5T was used throughout: this was deemed large enough to produce a useful Hall signal whilst also minimizing the quantum effects which will be discussed in a following section.

Figure 5.9 presents the (Hall) mobility as a function of the (Hall) sheet density at 350mK. Lower current drives yield results which are noisier but apparently better: this may be due to current heating effects. However, since this is a DC measurement with the current only flowing in one direction along the Hall bar, there may be an absolute offset in measured voltages which remains uncorrected and therefore leads to a systematic error.

Figure 5.7 A summary of the temperature dependence of the resistance of the device on wafer 55/53, at varying sheet density. High sheet densities (greater than  $3 \times 10^{11} \text{cm}^{-2}$ ) are “metallic” in that the resistance increases with temperature, and in these cases the resistance seems to saturate as temperature decreases. At the lowest density (of  $1.83 \times 10^{11} \text{cm}^{-2}$ ) the resistance is increasing weakly as temperature decreases.

Figure 5.8 The gate voltage dependence of the sheet carrier density in device 55/53, from the Hall effect at 350mK, using a field of 0.5T. Results from three different drive currents are shown: lower currents are noisier since voltage signals are smaller.

Figure 5.9 The relationship between mobility and sheet density (calculated using the Hall effect at 350mK with a field of 0.5T) for device 55/53. As in Figure 5.8, lower current drives produce noisier results, but here mobility also appears to increase (for a given sheet density) as current drive decreases.

Solid lines are  $\mu = n \frac{e}{p_s h}$  where  $n$  is an integer, the dotted line is for  $n = 1/2$ .

Since mobility is increasing with sheet density for the sheet density range explored, it is to be expected (from Chapter 3) that the dominant mobility-limiting scattering process is interface impurity scattering, with surface roughness beginning to have an impact towards the higher end of the range. However, the analysis described in Chapter 3 does not take into account the quantum corrections which are described in the following section so detailed calculations based on this are out of the question.

The 50nA and 100nA mobility results in Figure 5.9, for sheet densities greater than  $2.0 \times 10^{11} \text{cm}^{-2}$ , seem to show slight oscillations. (The noise in the 20nA results presumably masks them.) This may be a feature of the quantum nature of the charge transport which can only be seen at millikelvin temperatures, when the mobility has been measured at a high number of sheet density values.<sup>13,14</sup> Oscillatory features can also be seen in Figure 5.8.

### 5.5.3 Low-Field Magnetoresistance

As described in Section 4.1.4, at low temperatures (such that  $k_B T \ll E_F$ ) weak localization, particle-hole interactions (including the Zeeman effect) should generate characteristic low-field magnetoresistance behaviour.

In the absence of magnetic field, quantum interference effects will lead the path of a charge carrier to form a closed loop provided that the rate at which its direction changes around the loop (the elastic scattering rate  $\tau^{-1}$ ) is much greater than the rate at which its energy and therefore quantum phase changes (the dephasing rate  $\tau_\phi^{-1}$ ) and the coherence which leads to the quantum interference is broken. Part of the probability current of the charge carrying particle is localized in this loop, so the conductivity of the material is reduced.

The largest localization loops (essentially those which take a carrier  $\tau_\phi$  to circumscribe) cannot form in the presence of a small magnetic field, since the flux enclosed by the loop leads to a phase shift which ruins the constructive quantum

interference. As the magnetic flux density is increased, the maximum feasible loop size decreases until even the smallest localization loops (limited by the carrier requiring a few  $\tau$  to scatter around the whole loop) enclose too much flux. This causes the conductivity of the material to increase with magnetic field. Temperature is not an explicit feature of Equation 4.31 but the dephasing and elastic scattering times and also the transport scattering time (which features in the mobility, and therefore the diffusion constant) have temperature dependences which will be explored as the data is analyzed.

As the field continues to increase, the interaction correction due to Zeeman splitting between up and down spin states becomes comparable to  $k_B T$  (the up-spin and down-spin levels are resolved over thermal broadening) and the conductivity is reduced (Equation 4.37) by scattering between them.

Particle-hole interaction corrections to the conductivity (Equation 4.34) are temperature rather than field dependent, but would contribute slightly to the magnetoresistance when the corrected conductivity tensor is inverted. However, as seen from the data in Figure 5.7 and its discussion in a Section 5.5.1, interaction effects are not expected to be important.

Some results in this regime are shown in Figure 5.10. The gate voltage is such that the sheet density is  $1.8 \times 10^{11} \text{cm}^{-2}$ . At the lowest temperatures, the suppression of weak localization by the magnetic field causes the resistivity to fall to a minimum before other effects cause it to rise again. As temperature increases, the minimum in resistivity becomes weaker and moves to higher fields, and then is lost.

The zero-field resistivity  $\rho_0$  for this sheet density ranges from  $15 \text{k}\Omega$  at  $0.35 \text{K}$  to  $13 \text{k}\Omega$  at  $1.5 \text{K}$ : the fact that  $\rho_0$  is falling as temperature increases, coupled with its high value, means that the system is in a non-metallic regime. This can be characterized by

$$\rho_0 = \frac{h}{e^2} \frac{1}{k_F l} \quad 5.1$$

Figure 5.10 Low-field magnetoresistance of 55/53 at a sheet density of  $1.8 \times 10^{11} \text{cm}^{-2}$ , offset by the zero-field value of the resistance ( $15 \text{k}\Omega$  at  $0.35 \text{K}$  falling to  $13 \text{k}\Omega$  at  $1.46 \text{K}$  implying  $k_{FL} \sim 1.7$ ). The noisy lines are data, the smooth lines are fits to Equation 4.38 produced with the parameters given in Figure 5.11.

where  $h/e^2$  is the von Klitzing quantum resistance standard (25812.8 $\Omega$ ) and  $k_F l$  is the product of the Fermi wavenumber and the mean free path of the carriers. Here,  $k_F l \simeq 2$  but the truly metallic regime requires  $k_F l \sim 10$ .<sup>15</sup> The solid lines on Figure 5.9 represent constant resistivity values such that  $\mu = n \frac{e}{p_S h}$  where  $n$  is an integer. The dotted line is for  $n=1/2$ . This can be related to the metallic or insulating behaviour of the 2DHG, as described in Equation 5.1.

The zero-field resistivity has been subtracted from each of the datasets in Figure 5.10 so that the correction to the resistance (as a function of magnetic field) can be clearly seen for each temperature. Noisy lines are actual data, and smooth lines are fits generated using Equation 4.38 which encapsulates corrections to the zero-field resistivity due to weak localization, carrier interactions and the Zeeman effect at low fields. The fitting parameters which result are shown in Figure 5.11. (An effective mass of  $0.2m_e$  has been assumed throughout, on the basis of the Shubnikov-de Haas analysis in section 5.5.4. Generally, an increase in the assumed effective mass will lead to a proportional increase in the resultant relaxation rate from Equation 3.3.)

Four fitting parameters are shown in Figure 5.11, but  $\tau_r$  is actually found from the Hall mobility. The low-field negative magnetoresistance is caused by suppression of weak localization, so  $\tau$  and  $\tau_\phi$  are found from fitting this part of the curve.  $F^*$  is found from the positive magnetoresistance contribution (mainly due to the effect of Zeeman splitting on interactions) which can be seen when weak localization is suppressed. The Landé  $g$ -factor is fixed at 4.5.<sup>16</sup>

Figure 5.11 Fitting parameters used to create the smooth lines in Figure 5.10.

This fitting has been repeated at other sheet densities, and the results are summarized as follows: Figure 5.12 shows how the dephasing time  $\tau_\phi$  is roughly proportional to  $T^{-p}$ , where  $p \sim 1$  as can be seen in Table 5.1.<sup>17</sup>

$p_s/10^{11} \text{cm}^{-2}$	$p$ [ $\tau_\psi \propto T^{-p}$ ]
3.79	1.0±0.2
3.09	1.0±0.2
2.53	0.9±0.1
1.83	0.8±0.1
1.41	1.1±0.1

Table 5.1 Dephasing times obtained from fits (similar to those shown in Figure 5.10) were plotted against temperature for each sheet density in Figure 5.12. Fits to these data yield the index  $p$  which should be close to unity.<sup>17</sup>

Figure 5.13 shows how, for most sheet densities, the elastic scattering time  $\tau$  decreases with temperature. However, for the lowest sheet density (the pink curve) where  $k_F l \simeq 1$  and the 2DHG is on its way to becoming strongly localized,  $\tau$  decreases sharply with decreasing temperature. The mobility of the hole gas was found at each sheet density and temperature, employing the Hall effect as described in the preceding section. Using Equation 3.3, the transport scattering times  $\tau_r$  were obtained and plotted in Figure 5.14.

Lastly, the screening parameter  $F^*$  (defined in Equations 4.35 and 4.36) is shown in Figure 5.15. It is a cause for concern that  $F^*$  appears to vary with temperature: it is clear from its definition that it should only be a function of the sheet density (assuming factors such as the effective mass remain constant). In fact, its value should be at most 0.866 (in the limit of no carriers and therefore no screening) so the values returned from the fitting, being often greater than unity, are a sign of some deficiency in either the theory itself or its application.

Figure 5.12 The temperature dependence of the dephasing times summarized from data similar to that presented in Figure 5.10. The fits shown are for  $\tau_{\psi} \propto T^{-p}$  and values for  $p$  are given in Table 5.1.

Figure 5.13 A summary of the elastic scattering times from data similar to that presented in Figure 5.10.

Figure 5.14 These values of the transport scattering lifetime were calculated from the Hall mobility at a field of 0.5T, and are provided for comparison with Figure 5.12 and Figure 5.13.

Figure 5.15 Values of the renormalized screening parameter, summarized from data similar to that presented in Figure 5.10. According to its theoretical derivation,  $F^*$  can only take values between zero and  $\sim 0.866$ .<sup>19</sup>

The parameter  $F^*$  appears in both the temperature-dependent conductivity correction from interactions (Equation 4.34) and the magnetoconductivity correction from Zeeman splitting (Equation 4.37). Since these are both *conductivity* corrections, there is no reason to suggest that the forms used for  $F^*$  in each case should not be the same. (The conductivity screening parameter is sometimes referred to explicitly as  $\tilde{F}_\sigma$  whilst the form of the screening parameter used to calculate, for example, corrections to the specific heat  $\tilde{F}_C$  is different).<sup>18</sup> However, inconsistencies between the values of  $F^*$  produced by fitting different conductivity corrections to different results have been noted.<sup>12,19</sup>

#### Analysis of the Dephasing Time, $\tau_\phi$

For a 2-dimensional system, theory predicts that  $\tau_\psi \propto 1/T$  and that this should be generally independent of any other specific device parameters.<sup>20,21</sup> This is demonstrated in Table 5.1 and represented by the grey  $\hbar/\tau_\psi = k_B T$  line in Figure 5.10.

This result is more usefully expressed as  $\hbar/\tau_\psi \simeq k_B T / (k_F l_\psi)$  where  $k_F l_\psi$  is the dimensionless conductivity of the system if limited by dephasing alone (in the absence of magnetic field). However, since  $l_\psi = \sqrt{D \tau_\psi}$  then a self-consistent solution is:<sup>22</sup>

$$\hbar/\tau_\psi = \frac{k_B T}{k_F l} \ln(2k_F l) \quad 5.2$$

Dephasing times predicted in this case are larger than those in Figure 5.12 and if this were true then the data in Figure 5.12 for each sheet density should lie above the grey  $\hbar/\tau_\psi = k_B T$  line, generally further above it the greater the sheet density.

In any case, these forms predict a divergence in  $\tau_\phi$  as temperature decreases suggesting that at zero temperature there would be no dephasing at all, and no true

metallic behaviour. Experimental results seem to suggest that this is not the case, and  $\tau_\phi$  in fact saturates as temperature decreases.<sup>23</sup> Evidence for this can be seen most clearly in the results in Figure 5.12 for a sheet density of  $3.8 \times 10^{11} \text{cm}^{-2}$  (in black) where the data point at the lowest temperature value spoils the  $\hbar/\tau_\psi \propto k_B T$  fit. This could be due to heating or external electromagnetic noise effects,<sup>22</sup> or it may be the case that a finite dephasing rate even at zero temperature is fundamental and results in the familiar (but theoretically forbidden according to traditional scaling theories of localization) metal-insulator transition in two-dimensional systems.<sup>24</sup>

### Carrier Interactions

Two kinds of scattering are expected to contribute to the dephasing rate: (a) direct carrier-carrier interaction involving energy transfer of the order of  $k_B T$  and *large* momentum transfer, and (b) the interaction of one carrier with the time-fluctuating electromagnetic field (Nyquist noise) of all the other carriers, involving *small* momentum transfer.<sup>25</sup> The carrier-carrier relaxation rate is given by<sup>26</sup>

$$\frac{1}{\tau_{cc}} = \frac{F^2 \pi (k_B T)^2}{2 \hbar E_F} \ln \left( \frac{E_F}{k_B T} \right) \quad 5.3$$

where the term inside logarithm is replaced by  $\frac{E_F}{\hbar/\tau}$  if the carrier energy around the Fermi line is smeared by disorder rather than by temperature. The Nyquist relaxation rate is given by<sup>27</sup>

$$\frac{1}{\tau_N} = \frac{k_B T}{4 E_F \tau} \ln \left( \frac{2 k_B T}{\hbar/\tau_\psi} \right) \quad 5.4$$

(compare Equation 5.8) and in this case the term inside logarithm is replaced by  $\frac{E_F}{\hbar/\tau}$  in the special case that  $\tau_\psi = \tau_N$ . Mathiesen's rule (Equation 3.4) would then suggest

that:\*

$$\frac{1}{\tau_{\psi}} = \frac{1}{\tau_N} + \frac{1}{\tau_{cc}} \quad 5.5$$

When  $p_S=3.8 \times 10^{11} \text{cm}^{-2}$  at 1K, Equation 5.3 gives a carrier-carrier scattering time of 63ps. (The thermal energy  $k_B T$  is greater than the level broadening  $\hbar/\tau$  so the relevant cutoff is the thermal energy.) Equation 5.4 gives a Nyquist scattering time of 330ps. The total dephasing time from Equation 5.5 is 53ps, much greater than the measured dephasing time of 7.2ps. Calculations of the dephasing rate at other temperatures and sheet densities confirm that carrier-carrier and Nyquist scattering do not account for most of the dephasing.

This suggests that the actual, measured dephasing time  $\tau_{\phi m}$  is given by

$$\frac{1}{\tau_{\phi m}} = \frac{1}{\tau_N(\tau_{\psi})} + \frac{1}{\tau_{cc}} + \frac{1}{\tau_x(\tau_{\psi})} \quad 5.6$$

where  $(\tau_x)^{-1}$  represents the dephasing rate (which in general depends on the overall dephasing time due to lifetime broadening effects) from other processes. These other processes would seem to dominate in this device, contradicting the traditional theories which state that carrier-carrier scattering (in particular, electron-hole pair production) is the only way in which a carrier can lose energy at low temperatures in a semiconductor.

Two alternative theories follow, in which the dephasing rate is finite even at zero temperature.

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\* It may be argued that since the dephasing time is a function of itself then Mathiesen's rule should not strictly have meaning.

## Zero Point Fluctuations

It has been suggested that this saturation of the dephasing time in the limit of zero temperature is not due to heating effects or magnetic impurities (as had previously been suggested) but rather due to zero-point fluctuations of phase coherent electrons. This leads to<sup>23,28</sup>

$$\tau_{\psi} = \tau_0 \tanh \left( \alpha \pi \sqrt{\frac{\hbar / \tau_0}{k_B T}} \right) \quad 5.7$$

where  $\alpha$  is a constant of order unity and  $\tau_0$  is the zero-temperature saturation value of the dephasing time, which has so far been calculated only for the one-dimensional case.<sup>28</sup> Equation 5.7 apparently produces successful fits to data from a wide range of one and two-dimensional systems (with relevant modifications) once electron-phonon scattering is added  $\tau_{\psi ep} = [\tau_{\psi}^{-1} + \tau_{ep}^{-1}]^{-1}$  with a form  $\tau_{ep} = A_{ep} / T^3$ .<sup>23</sup>

This fits well to the  $p_s = 3.8 \times 10^{11} \text{cm}^{-2}$  (in black) data in Figure 5.12 with the values  $\tau_0 = 16.5 \pm 1.3 \text{ps}$ ,  $\alpha = 0.35 \pm 0.05$  and  $A_{ep} = 25 \pm 3 \text{psK}^3$ . This value for the strength of the electron-phonon interaction is anomalous: at the temperatures discussed here ( $\sim 1 \text{K}$ ) electron-phonon scattering is not important in silicon or silicon-germanium.\* The fact that in the limit of temperatures greater than  $\hbar / (k_B \tau_0)$  (0.4K in this case)  $\tau_0 \propto T^{-1/2}$  unless an extra term is added to Equation 5.7 would seem to be a problem. Fits to data at other sheet densities are less successful: the uncertainties on the returned parameters are greater than the values of the parameters themselves.

## High Frequency Fluctuations

Alternatively, the dephasing rate can be calculated within existing weak localization theory but incorporating high frequency ( $\omega \gg k_B T / \hbar$ ) quantum fluctuations to give:<sup>29</sup>

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\* See, for example, the acoustic-phonon limited mobility at 25K in Figure 5.28

$$\frac{1}{\tau_\psi} = \frac{e^2}{2h\sigma\tau} \left[ 1 + 2 \frac{k_B T}{\hbar l \tau} \ln \left( \frac{k_B T}{\hbar l \tau_\psi} \right) \right] \quad 5.8$$

The saturation value of the dephasing time in this case is therefore (from Equation 5.1) simply  $\tau_0 = 2k_F l \tau$ . For  $p_s = 3.8 \times 10^{11} \text{cm}^{-2}$ , inspection of Figure 5.13 suggests that a zero temperature elastic scattering time of 1.2ps is reasonable, and since  $k_F l = 8$  at this sheet density then  $\tau_0$  is found to be  $\sim 20$ ps. At other sheet densities, though, a good value for  $\tau(T=0)$  seems less obvious. The temperature dependence of Equation 5.7 is weaker than  $1/T$  and, as with Equation 5.7, electron-phonon scattering could be explicitly added. However, as noted below, since the dephasing time is a function of itself in Equation 5.8 then Mathiesen's rule (Equation 3.4) may not strictly be applicable. Equations 5.2, 5.7 and 5.8 are compared in Figure 5.16.

Whether it is necessary to modify the fundamentals of weak localization theory is controversial.<sup>30,31,32,33</sup> It is argued that the zero-point fluctuations which lead to Equation 5.7 (or the unoccupied high-frequency modes which lead to Equation 5.8) cannot cause any dephasing, since the energy of a harmonic oscillator may not be available to be transferred in situations where the level spacing is much greater than  $k_B T$ , and that the saturation of the dephasing time is due to *external* electromagnetic noise of microwave frequencies.<sup>32</sup> It is further suggested that only particle-particle scattering is relevant to dephasing (the Coulombic interaction being an elastic process) and any finite dephasing at zero temperature predicted theoretically is based on a profoundly incorrect calculation.<sup>22</sup> Alternatively, it is argued that the zero-temperature dephasing time is lost in calculations which are performed incorrectly.<sup>33</sup>

The form of the mobility as a function of sheet density (Figure 5.9) and the calculations described in Chapter 3 suggest that the scattering that limits the transport is due to screened interface impurities (at low sheet densities) and surface roughness (as sheet density increases) plus quantum effects which will be discussed in Section 5.6.2.

Figure 5.16 A summary of dephasing times, calculated by various theories and compared to data from Figure 5.12. The AAG line is based on Equation 5.2. MJW was produced by fitting Equation 5.7 including phonons, but as the text suggests requires unreasonably strong electron-phonon scattering; MJW(2) is Equation 5.7 alone. Equation 5.8 was used to produce the GZ line.

These contribute only to elastic scattering, and therefore momentum relaxation, rather than the dephasing through inelastic processes. (It is puzzling, though, that the elastic scattering times seem to be 2 or 3 times larger than the momentum relaxation (transport) time). Recent experimental studies confirm that in high mobility silicon-germanium systems, scattering is dominated by potential fluctuations and the loss of screening rather than by carrier-carrier interactions.<sup>34</sup>

With finite elastic scattering rates but no dephasing, at zero temperature a 2D system would be completely localized. The tendency of the resistivity to saturate at a finite value as temperature decreases, for large enough sheet density, contradicts this. (One caveat is that insulating behaviour could set in at a temperature an order of magnitude lower than has been explored in this study.)

The implication is that the dephasing rate is proportional to the sheet density: in high density systems which remain metallic as temperature decreases, the dephasing rate must be prevented from continuing to increase as  $T^{-1}$ . In Equations 5.3 and 5.4, the dephasing time is proportional to the Fermi energy, meaning that high density systems would have longer dephasing times than low density systems. The Nyquist scattering rate (Equation 5.3) also incorporates the elastic scattering time which increases as impurities are screened by an increasingly dense carrier gas. The carrier-carrier scattering rate (Equation 5.4) incorporates the screening parameter  $F$  which decreases as sheet density increases, increasing the carrier-carrier scattering time further.

The alternative theories presented, which give a finite dephasing rate at zero temperature from either zero-point or high-frequency quantum fluctuations, do not seem to reproduce the  $T^{-1}$  result in any limit. The density dependence of the dephasing time (due to zero-point fluctuations) in Equation 5.7 is presumably incorporated into  $\tau_0$  which remains uncalculated for 2D systems. Equation 5.8 does ensure that the dephasing time (due to high-frequency fluctuations) increases with the conductivity of the system.

### 5.5.4 High-Field Magnetoresistance

The magnetoresistance of the device in the high-field, low temperature regime (where  $\mu B \sim 1$  and  $\hbar \omega_c \sim k_B T$  as described in section 4.1.3) is shown in Figure 5.17. The former shows conventional Shubnikov-de Haas oscillations and the Quantum Hall effect. A filling factor of unity is reached at the highest available fields. At low fields, minima in resistivity appear at odd values of the filling factor. Spin splitting occurs at around 5T, so  $\nu=2$  is also a minimum for all sheet densities but  $\nu=4$  only features a weak minimum at the highest sheet density.

For lower sheet densities than Figure 5.17, where the 2DHG is insulating in character, Figure 5.18 shows how a high resistance state appears between filling factors 1 and 3. This can be compared with the last  $\rho_{xx}$  maximum visible in Figures 5.17 and 5.18 (where the 2DHG is metallic) which is not particularly pronounced. The horizontal scale is linear in applied field but scaled so that the low-field oscillations coincide:

$$\frac{1}{\nu} = \frac{eB}{n_{SDH} h} \quad 5.9$$

where  $\nu$  is the filling factor described in section 4.1.3. In general, the sheet density  $n_{SDH}$  found through Equation 5.9 will not perfectly match that found from the low-field Hall effect. This can be seen in Figure 5.19.

The magnetic-field-induced metal-insulator transition is well known in bulk semiconductors where it is caused by squeezing of the electronic wavefunction at localization centres.<sup>35</sup> In a 2-dimensional system the off-diagonal conductivity  $\sigma_{XY}$  is proportional to the filling factor but a divergence in  $\rho_{xx}$  implies vanishing  $\sigma_{XY}$ . This means that the large maximum in  $\rho_{xx}$  at a filling factor of  $\sim 1.6$  cannot be explained in terms of the Quantum Hall Effect.

Figure 5.17 Magneto-resistance of 55/53 in the low temperature, high field regime where Shubnikov-de Haas oscillations and the Quantum Hall Effect are visible. Sheet densities (calculated from the Hall effect) are such that the 2DHG is metallic in character. In general, the sheet density calculated from the positions of the minima and maxima in the oscillations will not exactly match that from the Hall effect.

Figure 5.18 Magnetoconductance of 55/53 in the low temperature, high field regime where sheet densities are such that the 2DHG is insulating in character. The vertical axis has been rescaled by the magnitude of the zero-field resistance for each sheet density: in absolute terms, the resistance peak for the lowest sheet density is of the order of  $1\text{M}\Omega$ , nearly two orders of magnitude greater than the peak in the highest sheet density (just over  $20\text{k}\Omega$ ). The horizontal axis has been scaled using the Shubnikov-de Haas rather than the Hall sheet density.

Figure 5.19 A comparison of the sheet density extracted from the Hall effect (from Figure 5.8) with that found from Shubnikov-de Haas oscillations.

This has been taken as evidence for Wigner crystallization, but if  $\nu > 1$  then at least one totally occupied magnetic level should exist below the Fermi level with a band of delocalized states located at the magnetic level centre, giving a non-zero  $\sigma_{XY}$ .<sup>35,36</sup>

The insulating phase between filling factors of 1 and 2 has been seen in p-type silicon-germanium systems<sup>37,38,39,40</sup> but is not predicted in the global phase diagram for 2D systems.<sup>41</sup> Magnetic-field-induced phase transitions have been observed in the GaAs/AlGaAs system, but insulating states only tend to form at  $\nu \ll 1$ .<sup>42,43,44,45</sup> The unusual energy level degeneracy in p-type silicon germanium may be central to its formation.<sup>40</sup>

More recently, it has been suggested that the similar transition from the insulating state to the metallic state in silicon MOSFETs at zero field has a percolation nature, indicating electron localization to be the origin of the former state.<sup>35,37,46,47</sup>

Figure 5.20 shows how Shubnikov-de Haas oscillations (in the region where the field is low enough to avoid spin-splitting) decay with temperature. As described in Section 4.1.3, this data can be used to find the effective mass  $m^*$  of the carriers, and also the ratio between the quantum and transport scattering lifetimes,  $\alpha = \tau_{tr} / \tau_q$  (from Equation 4.26). However, analysis of these results (or results at other sheet densities) does not yield self-consistent values for  $m^*$  or  $\alpha$ . With  $\alpha \approx 1$  the plot described in Equation 4.28 gives  $m^* \approx 0.3 m_e$ . The gradient of the plot described in 4.30, however, yields  $\alpha = 0.7$  and if the iterative process is continued then the values of both  $m^*$  and  $\alpha$  diverge.

Figure 5.20 Shubnikov-de Haas oscillations as a function of temperature, for fields low enough to avoid spin-splitting. Sheet density from the Hall effect is  $3.79 \times 10^{11} \text{cm}^{-2}$  but from the period of the oscillations is  $4.22 \times 10^{11} \text{cm}^{-2}$ . The drive current is nominally 50nA.

## 5.6 Results from the Siemens Device

### 5.6.1 Room-Temperature IV Characterization

Room temperature measurements of this device, shown in Figure 5.21, demonstrate p-channel MOSFET-like behaviour at drain-source voltages generally small enough to avoid pinch-off. The threshold voltage  $V_{TS}$  is around -1.0V: in this region, at higher  $V_{DS}$ , the drain current increases quadratically as the gate-source voltage becomes more negative. Away from the threshold, the drain current is linear with both gate-source voltage and drain-source voltage.

### 5.6.2 Mobility as a Function of Sheet Density and Temperature

The results shown in Figure 5.22 and Figure 5.23 were taken during preliminary tests of a device, using the HP parameter analyzer and closed-cycle cryostat. (A constant drain-source potential of 10mV was applied, whereas the usual method is to apply a constant drain current. This means that the drain current was of the order of a few microamps at times, which is rather high.)

Later results are worse in terms of the actual peak mobility value as can be seen from the 350mK results shown in Figure 5.24 and Figure 5.25:  $1700\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $1.3 \times 10^{12}\text{cm}^{-2}$  as opposed to  $3300\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $1.2 \times 10^{12}\text{cm}^{-2}$  at 10K in Figure 5.23. (A current of 10nA was used for these measurements, to avoid heating the carriers above the lattice temperature.<sup>10</sup>) Whilst this mobility does not seem impressive compared with some of the best in p-type strained silicon-germanium alloys (for example,  $15000\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $1.8 \times 10^{11}\text{cm}^{-2}$  at 0.35K in a normally-doped  $\text{Si}_{0.87}\text{Ge}_{0.13}$  alloy grown at  $950^\circ\text{C}$ )<sup>48</sup> it is worth noting the relatively high sheet density (giving a sheet resistivity of  $2.6\text{k}\Omega/\square$ ) and high germanium content of the pseudomorphic alloy layer.

Figure 5.21 Room temperature IV results for the Siemens device. The gate-source threshold Voltage  $V_{TS}$  is around -1.0V. Drain current rises linearly with drain-source voltage for gate-source voltages more negative than this. Behaviour is consistent with that of a p-channel MOSFET.

Figure 5.22 Early Hall-effect results at low temperature: sheet density increases linearly as the gate voltage becomes more negative, but the linearity breaks down eventually. (The gate capacitance is  $\sim 0.2\text{pF}$ )

Figure 5.23 More early Hall-effect results, showing typical behaviour for the mobility as a function of sheet density.

Figure 5.24 Hall-effect results at 350mK (with a drain current of 10nA) showing the dependence of sheet density on gate voltage. The good agreement between the results at two different values of magnetic field suggest either that the Hall results are very close to the true ("drift") results, or that the mobility of the 2DHG is very small. From this, the capacitance of the gate is  $\sim 0.3\text{pF}$ . (The gate voltage was swept in both directions, and the results averaged, for each field.)

Figure 5.25 The mobility at 350mK peaks at more than  $1700\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  but this value seems worse than that seen in Figure 5.23. Again, values at two different fields match closely.

A 4K mobility of  $2500\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $6.2\times 10^{11}\text{cm}^{-2}$  (and therefore a sheet resistivity of around  $4\text{k}\Omega/\square$ ) has been reported in a high-quality pseudomorphic  $\text{Si}_{0.64}\text{Ge}_{0.36}$  p-channel device,<sup>49</sup> 77K hole mobilities of up to  $3500\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $3\times 10^{12}\text{cm}^{-2}$  have been reported in a system with a  $\text{Si}_{0.2}\text{Ge}_{0.8}$  channel grown on a virtual substrate.<sup>50</sup>

Some sort of noise can be seen in Figure 5.24, especially at the low and high sheet density ends of the  $\pm 1\text{T}$  data. This may be related to the oscillations seen in Figure 5.8 and Figure 5.9, discussed in Section 5.5.2. The off-diagonal component of the resistivity tensor,  $\rho_{XY}$ , is shown in Figure 5.26 as a function of sheet density (assuming a linear dependence on gate voltage as in Figure 5.24). The relationship

should be  $\rho_{XY} = \frac{B}{p_S q}$  (Equation 4.6) but quasi-oscillatory features can be seen. These features clearly do not depend on the magnetic field (so are not, for example, due to the formation of Landau levels) and are repeatable (so are fluctuations as a function of gate voltage, not time). These fluctuations in  $\rho_{XY}$  may be a sign of fluctuations in  $\rho_{XX}$ ;<sup>13,14</sup> no fluctuations were visible in the  $\rho_{XX}$  data, but this may be because the absolute value of  $\rho_{XX}$  tended to be much larger than the scale of the fluctuations, drowning them out. These oscillations are largely suppressed in Figure 5.24 since they cancel out when positive and negative field results are combined.

Comprehensive results for the mobility as a function of sheet density from 350mK to 282K are presented in Figure 5.27. (There is essentially no variation between 350mK and 1.4K.) These are the subject of the following calculations.

### Calculations of Mobility as a Function of Sheet Density

In Chapter 3, the mobility of a 2-dimensional carrier gas as a function of sheet density is discussed in terms of the mechanisms which may limit it. Calculations have been performed in similar studies.<sup>51,52</sup>

Figure 5.26 From Equation 4.6,  $\rho_{XY}$  should be inversely proportional to  $p_S$  at a constant field. The x-axis has been scaled from gate voltage to sheet density using a straight-line fit to Figure 5.24, but the data clearly show anomalous quasi-oscillatory features. Importantly, these features appear not to depend on the magnetic field.

Figure 5.27 Hall mobility as a function of sheet density, using magnetic fields of  $\pm 1\text{T}$ , at temperatures from 350mK up to almost room-temperature. Lines are shown at  $\mu = 2^n \frac{e}{p_s h}$  with solid lines at  $n=0,1,2,3$  and 4 and dotted lines at  $n=-1$  and -2.

Experimental data at 25K is presented in Figure 5.28, along with calculated values for the mobility produced using a C program written by A. I. Horrell\* using the forms presented in References 51 and 52. The mobility limit set by each scattering process is shown (apart from optical phonons, which are off the scale at this temperature) so that it is clear how interface impurities limit the mobility at low sheet densities and interface roughness becomes dominant as sheet density increases. At the lowest sheet densities, where the carriers are becoming increasingly localized, multiple scattering may need to be taken into account.<sup>53</sup>

The parameters used to produce these calculated mobilities are shown in Table 5.2. Those in italics were taken from the literature and held constant whilst those in normal type were varied to create a fit to the experimental data, based on reasonable initial values from the literature.

The fit was performed at 25K because this temperature is well below  $T_F$  in this sheet density range (meaning that the carrier gas is degenerate and therefore that the Hall mobility needs no correction due to energy-dependent scattering mechanisms) but should be high enough that the transport is not be influenced by any of the factors described in Section 4.1.4.

The fit in Figure 5.28 is quite successful, demonstrating that the factors which limit the mobility are those which are open to improvement through better growth. Fundamental limits on mobility set by alloy scattering, for example, are several times greater than those measured. However, Figure 5.29 shows how calculations of the mobility at other temperatures measure up (with these same parameters) against experimental data and the agreement is less successful.

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Figure 5.28 Calculated mobility at 25K, showing contributions from each of the relevant mechanisms: It can be seen that interface impurities limit the mobility at the lowest sheet densities, with surface roughness becoming dominant over most of the rest of the range. The optical-phonon-limited mobility is off the scale.

Figure 5.29 Comparison of mobility calculated and measured across the whole temperature range. The parameters used for the calculation are those from the fit at 25K, shown in Table 5.2.

Depletion charge density <sup>12,51</sup>	$N_D$	$20.0 \times 10^{11} \text{cm}^{-2}$
Interface impurity charge density <sup>12,51</sup>	$n_i$	$2.8 \times 10^{11} \text{cm}^{-2}$
<i>Effective mass in the growth direction</i> <sup>54</sup>	$m_z$	$0.3 m_e$
<i>Effective mass for in-plane transport</i> <sup>54</sup>	$m^*$	$0.3 m_e$
<i>Relative permittivity</i> <sup>54</sup>	$\epsilon_r$	$14.0 \epsilon_0$
Interface roughness height <sup>51</sup>	$\Delta$	0.46 nm
Interface roughness correlation length <sup>51</sup>	$\Lambda$	1.20 nm
<i>Alloy scattering interaction strength</i> <sup>51,52</sup>	$\delta E$	0.6 eV
<i>Lattice mismatch factor</i> <sup>51,55</sup>	$f$	0.0125
<i>Poisson's ratio</i> <sup>51</sup>	$\nu$	0.28
<i>Acoustic phonon deformation potential</i> <sup>10,51,56</sup>	$\Xi_u$	4.5 eV

Table 5.2 Parameters used in the calculations presented in Figures 5.28, 5.29 and 5.30. Those in italics were taken from the literature and held constant whilst those in normal type were varied to create a fit to the experimental data, based on initial values from the literature.

Using Equation 3.5 and the value of effective mass quoted in Table 5.2, the Fermi temperature in Kelvin is given by:

$$T_F = 93 p_s \quad 5.10$$

with  $p_s$  in units of  $10^{12} \text{cm}^{-2}$ .

This means that, for example, at 152K the carrier gas would become non-degenerate as the sheet density decreases below  $1.6 \times 10^{12} \text{cm}^{-2}$  and the mobility and sheet density measured with the Hall effect would become increasingly affected by energy-dependent scattering and band-structure anisotropy.

The data and calculated values in Figure 5.29 at 152K do indeed coincide reasonably well at high sheet densities (bearing in mind that no attempt has been made to fit any of the parameters in Table 5.2 which directly relate to the temperature dependence of the mobility) but diverge as the sheet density drops below around  $2 \times 10^{12} \text{cm}^{-2}$ . At 282K, where the carrier gas is more or less non-degenerate throughout the whole (Hall) sheet density range (for a  $T_F$  of 282K, according to Equation 5.10,  $p_s$

would have to be  $3.0 \times 10^{12} \text{cm}^{-2}$ ) the Hall scattering factor can be found. From Equation 4.8,  $\sigma = p_{\text{Hall}} q \mu_{\text{Hall}} = p_{\text{Drift}} q \mu_{\text{Drift}}$  so

$$r = \frac{p_{\text{Drift}}(\sigma)}{p_{\text{Hall}}(\sigma)} \quad 5.11$$

This analysis yields a Hall scattering factor at 282K of  $0.68 \pm 0.04$ , averaged over the sheet density range. The influence of energy-dependent scattering can only lead to Hall scattering factors greater than unity (Equation 4.19); this value is consistent with other work in the field and is suggestive of an anisotropic Fermi surface as is believed to be the case for hole transport in strained silicon germanium alloys.<sup>57,58,59</sup>

The value found in this work is actually slightly larger than values found elsewhere, but this may be due to additional contributions from energy-dependent scattering mechanisms. In Reference 59 the Hall scattering factor is found to generally increase with sheet density from around 0.4 at  $3 \times 10^{11} \text{cm}^{-2}$  to 0.8 at  $4 \times 10^{12} \text{cm}^{-2}$ . (This behaviour is independent of the composition of the alloy layer.) The tendency of the Hall factor to unity as sheet density increases is unsurprising, since the Fermi temperature is directly proportional to the sheet density; as sheet density increases (at any given temperature) the carrier gas will gradually become degenerate. The Hall scattering factor is unity when  $T_F \gg T$ .<sup>58</sup>

From Equations 4.20 and 5.11, the 282K drift mobility of this device is therefore around  $350 \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$  at  $10^{12} \text{cm}^{-2}$ . A drift mobility of  $330 \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$  at  $5 \times 10^{11} \text{cm}^{-2}$  has been measured in a 6nm-thick modulation-doped p-type  $\text{Si}_{0.6}\text{Ge}_{0.4}$  alloy layer at 295K;<sup>59</sup> for comparison, in a pure germanium channel grown on a  $\text{Si}_{0.3}\text{Ge}_{0.7}$  virtual substrate a mobility at room temperature of  $1700 \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$  at  $2.3 \times 10^{12} \text{cm}^{-2}$  has been recently reported.<sup>60</sup>

At 10K and 0.35K, once again the calculations fail to fit to the experimental data for mobility as a function of sheet density. In Figure 5.30 experimental data for the

resistivity as a function of sheet density is compared to calculations at certain sheet density values, and the form fits quite badly. Clearly, there are effects which augment the mobility of metallic carrier gases and diminish the mobility of insulating carrier gases, which are not included in the method and limit its application at low temperatures where these effects are significant. This will be explored in the following section.

### 5.6.3 Resistance as a Function of Temperature and Sheet Density

Figure 5.31 shows how the resistivity of the Siemens device varies as a function of temperature at a few different sheet carrier concentrations around the peak in mobility. The resistivity appears to be saturating as the temperature decreases; for low sheet densities resistivity increases with decreasing temperature (insulating behaviour) but for higher densities it decreases (metallic behaviour).

Resistivity (or conductivity) at ~1K is usually discussed in terms of weak localization, interactions (Sections 4.1.4, 5.5.3 and 5.6.4) and screening.<sup>12,61</sup> Equation 5.12 is based on finite-temperature screening theory, if  $k_B T \ll \hbar/\tau$  and  $T < T_F$ .<sup>62</sup>

$$\sigma(T) = \sigma(0) \left[ 1 - \beta \left( \frac{T}{T_F} \right)^2 \right] + \frac{A e^2}{\pi \hbar} \ln \left( \frac{k_B T}{\hbar/\tau} \right) \quad 5.12$$

And Equation 5.13 on the temperature dependence of the static dielectric function:<sup>63</sup>

$$\sigma(T) = \sigma(0) \left[ 1 - C \frac{T}{T_F} \right] + \frac{A e^2}{\pi \hbar} \ln \left( \frac{k_B T}{\hbar/\tau} \right) \quad 5.13$$

Figure 5.30 Resisitivity as a function of temperature, comparing experimental data with calculations similar to those demonstrated in Figure 5.28 and Figure 5.29.

Figure 5.31 Temperature dependence of the resistivity around the metal-insulator transition. At the lowest sheet density shown, resistivity increases as temperature decreases but may be saturating. At larger sheet densities, the resistivity decreases to a saturation value as the temperature decreases. (The behaviour changes little as sheet density continues to increase.)

The second term in Equations 5.12 and 5.13 is the correction to the conductivity due to interactions, Equation 4.34;<sup>64</sup>  $\tau$  is the elastic scattering time which in this system should be roughly equal to the momentum relaxation (transport) lifetime and  $A = \left(1 - \frac{3}{4}F^*\right)$ . The constants  $\beta$  and  $C$  should be of order unity, and  $\sigma(0)$  is the zero-temperature Boltzmann conductivity. If the low-field magnetoresistance showed any evidence for weak-localization, then a term for the temperature dependence of the dephasing would be incorporated in  $A$ .<sup>12,61</sup>

However, for sheet densities of the order of  $10^{12}\text{cm}^{-2}$  and an effective mass of the carriers around  $0.3m_e$  the Fermi temperature  $T_F$  (Equation 3.5) is over 100K, a factor of three greater than in Reference 12.

For temperatures which are high (but still such that  $T < T_F$ ) the interaction term contribution weakly increases with temperature but the first term in both Equations 5.12 and 5.13 dominates, causing the conductivity to decrease. As the temperature decreases toward zero, however, the interaction term dominates and the conductivity decreases. Hence, there should be a maximum in the conductivity and this is indeed seen in References 12 and 61 (at 2K and 0.8K respectively) where forms similar to Equations 5.12 and 5.13 are successfully applied. (Eventually the interaction term in Equations 5.12 and 5.13 breaks down, tending to negative infinity: in this limit,  $k_B T$  must be replaced by a temperature-independent upper cut-off.<sup>65</sup> This avoids an unphysical negative overall conductivity.<sup>66</sup>)

A maximum in conductivity (that is, a minimum in resistivity) may be seen in Figure 5.31 in the data at a sheet density of  $1.07 \times 10^{12}\text{cm}^{-2}$  but not at other sheet densities, and the resistivity is clearly not increasing as temperature decreases further. Analogous data in device 55/53 (Figure 5.7) also fail to clearly show a minimum in resistivity at metallic sheet density values. Another approach is needed to evaluate these data.

Work on the metal-insulator transition in 2-dimensional samples suggests that the resistivity can be approximated by the following expression:<sup>15,67</sup>

$$\rho(T) = \rho_0 + \rho_1 \exp[-(T_0/T)^n] \quad 5.14$$

$\rho_0$ ,  $\rho_1$  and  $T_0$  are dependent on sheet density but not temperature. In p-type silicon germanium heterostructures, good fits have been achieved with  $n \approx 0.5$ . The exponential term becomes decreasingly important as sheet density increases.<sup>67,68</sup>

In order to fit the data in Figure 5.31, the following form for  $\rho(T)$  is introduced:

$$\rho(T) = \rho_0 + \rho_c \exp[-(T/T_0)^n] + \rho_l \ln \left[ 1 + \frac{k_B T}{\hbar / \tau} \right] \quad 5.15$$

The first term, which is constant with respect to temperature, represents elastic scattering from impurities<sup>18</sup> and can be estimated using  $\rho_0 = (p_s q \mu_0)^{-1} - \rho_c$  where  $\mu_0 = \mu(T \rightarrow 0)$ . (The possibility of metallic transport at zero temperature is discussed in section 5.5.3, in the context of quantum dephasing mechanisms at zero temperatures which may play a role in the metal-insulator transition.<sup>69,70</sup>)

The second term represents a thermally activated process which enhances the conductivity as temperature increases. An exponential form is used, but since the resistivity does not vary by more than an order of magnitude this may not be necessary (and the very high conductivity state which may feature a re-entrant insulator-metal-insulator transition as seen in gallium arsenide<sup>71</sup> is never reached). In any case,  $1 \geq n > 2$  keeps this term well behaved as  $T \rightarrow 0$ .

The third term appears superficially similar to the interaction correction discussed above, but in this case causes the resistance to rise weakly with temperature. The use of the  $\ln(1+x)$  form again ensures good behaviour as  $T \rightarrow 0$ , and  $\ln(1+x) \approx x$  for low  $x$ .

Fitting parameters are summarized in Figure 5.32:  $n$  is fixed at 1.5. At low sheet densities, for the temperature range considered, the third term in Equation 5.13 becomes redundant and Equation 5.14 is recovered.

While this form successfully fits the data, it has no direct theoretical justification and the behaviour of  $\rho(T \rightarrow 0)$  at high sheet densities remains unresolved. Low-field magnetoresistance may clarify the influence of quantum corrections to the conductivity,<sup>72</sup> as it has in the case of the previously discussed devices.

#### 5.6.4 Magnetoresistance

For temperatures between 25K and 300K, magnetoresistance data (taken across the sheet density range shown in Figure 5.24) shows an essentially constant  $\rho_{xx}$  and  $R_H$  up to 11T. This makes it clear that, firstly, no weak localization or Landau level formation is present at these temperatures (which is to be expected) and secondly that there is no parallel conduction in, for example, an inversion layer at the interface between the silicon cap layer and the silicon dioxide gate dielectric.

The form of the 350mK magnetoresistance data in Figure 5.33 contrasts strongly with that of Figure 5.17 and Figure 5.18. The magnetoresistance is always positive, showing no evidence of the low-field negative magnetoresistance that is the signature of weak localization. Features at high field in some of the data at lower sheet densities may be related to the formation of Landau levels (at  $10^{12}\text{cm}^{-2}$ , 8T corresponds to a filling factor of 5) but  $\rho_{xy}$  (not shown) does not show signs of Quantum Hall plateaux. High field magnetoresistance may be due mainly to the effect of Zeeman splitting on interactions. Spin-orbit scattering gives rise to positive magnetoresistance at low fields.<sup>34,73</sup> Data taken by other workers on another device from the same Siemens wafer is shown in Figure 5.34. Shubnikov-de Haas oscillations are clearly visible, but the device was destroyed before more data at other temperatures and gate voltages could be obtained.

Figure 5.32 A summary of parameters for fitting Equation 5.15 to the data in Figure 5.31: at low sheet densities the third term does not contribute at these temperature so  $\rho_l \rightarrow 0$  and  $\tau$  is undefined.

Figure 5.33 Magnetoresistance data at 350mK, with the zero field value of the resistance subtracted in each case. The magnetoresistance is always positive, in contrast to Figure 5.17.

Figure 5.34 This magnetoresistance data was obtained during an early phase of device characterization by other workers. There are clear Shubnikov-de Haas oscillations at this temperature (1.47K) but the device was destroyed before further measurements could be performed.

It is not strictly possible to apply the analysis described in section 4.1.3 when data at only one temperature are available; the effective mass is normally found from the temperature decay of the oscillations. However, the variation of amplitude of the oscillations with respect to magnetic field can be used to find  $\alpha$  if an effective mass is assumed. In fact, if Equation 4.28 is plotted as described then the data points representing the minima and maxima at this temperature should all lie on the line  $y = x + \ln(4)$  if the choices of  $m^*$  and  $\alpha$  are correct. It was found that the best fit to a line of unity gradient from a plot of Equation 4.28, with a consistent value of  $\alpha$  from the gradient of the plot of Equation 4.29, was produced with  $m^* = 0.3m_e$  and  $\alpha = 3$ . However, the intercepts of both of these plots was much greater than the value  $\ln(4)$  and attempts to correct this destroyed the consistency between the two plots. In other words, the oscillations are much larger in amplitude than the theory allows: the numerical factor before the summation in Equation 4.24 would have to be several times larger and there is no theoretical reason for this to be the case.

## 5.7 Conclusions

### 5.7.1 55/53

This device has proved to be an interesting and useful testbed for aspects of 2-dimensional physics. Its resistivity as a function of temperature weakly saturates at finite (sheet density dependent) value as temperature drops to zero, which would seem to contradict conventional theories of 2-dimensional transport based on scaling described in terms of the results from the Siemens device.

Whilst the mobility values measured were themselves impressive, fluctuations can be seen in Figure 5.8 and Figure 5.9 which as yet remain unidentified: they may or may not be related to the effects seen in the Siemens device described in section 5.5.2 and References 13 and 14. Generally, data for the mobility as a function of sheet density is not as detailed as that presented in Figure 5.9 so subtle features will not be seen.

The low-field magnetoresistance seems to be explained well within the framework of weak-localization; the contribution from interactions and Zeeman splitting, however, is less clear due to the anomalous values of the screening parameter  $F^*$  returned by the fitting process. The temperature dependence of the dephasing time was considered in terms of traditional weak-localization, where the dephasing rate drops to zero at zero temperature, and controversial new ideas which suggest that the dephasing rate remains finite at zero temperature. The results are inconclusive, in that the traditional theory tends to predict dephasing times which are too long but the newer ideas do not fit the data very well unless unreasonable freedom is given to their free parameters. The question of finite dephasing at zero temperature is important and fundamental.<sup>74</sup> It is suggested that the theory most needs modification so that the dephasing rate increases with increasing carrier density, leading to metallic behaviour at high densities and low temperatures.

The behaviour of the elastic scattering time at low temperatures, however, does not appear to be of such theoretical interest even though it is of more direct relevance to the momentum relaxation time and therefore the mobility. From Figure 5.13, the elastic scattering time seems to be generally increasing with decreasing temperature in all but one case.

High-field magnetoresistance in the metallic regime is conventional, but for lower sheet densities a magnetic-field-driven transition from the quantum Hall effect state to an insulating state with a very high resistivity occurs at a filling factor  $\sim 1.6$ . This is often seen in silicon and silicon-germanium systems, and is related to the zero-field metal-insulator transition seen in gated devices. Both this and the results of low-field magnetoresistance characterization may contribute to the understanding of localization and interactions in 2-dimensional semiconductors, and therefore the metal-insulator transition and the nature of mobility-limiting scattering mechanisms.

### 5.7.2 *Siemens Device*

It is clear from the long term characterization of Siemens devices that results

during the earliest phases were more promising than those from the main investigation detailed in this chapter. However, during these early phases devices would last no more than a few days before they would be destroyed by stray voltages; only the protection circuit described in section 5.3.2 has made it possible to perform systematic and detailed characterization of a single device. As research focus moves towards the characterization of devices for commercial applications such considerations will become increasingly important.

It may be the case that the drop in performance between Figure 5.23 and Figure 5.25 is due to the full or partial relaxation of the active channel. Results obtained by H. E. Fischer at Siemens<sup>75</sup> at 4K show a peak mobility of  $1800\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $5\times 10^{11}\text{cm}^{-2}$  (the peak mobility in Figure 5.25 is  $1700\text{cm}^2\text{V}^{-1}\text{s}^{-1}$  at  $1.3\times 10^{12}\text{cm}^{-2}$ ). The fact that this peak is at a lower sheet density in the data of H. E. Fischer than in Figure 5.25 suggests that interface impurities are a comparatively serious issue in the device studied here.<sup>51</sup>

The peak mobility in Figure 5.25 is consistent with calculations for a relaxed  $\text{Si}_{0.5}\text{Ge}_{0.5}$  alloy.<sup>76</sup> On research-scale devices this can be verified by X-ray or micro-Raman measurements, or transmission electron microscopy. However, the small scale of the devices makes such analysis impossible in this case.

Degradation of the oxide may instead be the cause of the drop in performance. Since the carrier gas exists in the alloy layer only a few nanometres away from the oxide interface, the quality of this interface can limit the hole mobility in the strained channel.<sup>77</sup> Experience suggests that ESD damage is catastrophic rather than progressive, so this is not the cause. Ionic contamination (with, for example,  $\text{Na}^+$ ) would lead to shifting of the threshold voltage and possibly changes in the transconductance characteristics,<sup>2</sup> which were not seen. A remaining possibility is that the oxide interface has been degraded by hot charge carriers.<sup>78,79,80</sup>

It is possible, also, that the small size of the device is itself the issue: a 5nm  $\text{Si}_{0.5}\text{Ge}_{0.5}$  alloy grown on and capped with pure silicon should remain fully strained (up

to at least 500°C)<sup>7</sup> but the small lateral scale of the mesa (the Hall bar is 2.5μm wide) may have lead to significant relaxation towards the edges of the device.<sup>81</sup> This would have the strongest implications for high-field quantum magnetotransport and may explain the absence of Shubnikov-de Haas oscillations in Figure 5.33 (since conduction is concentrated towards the edges in the regime where  $\hbar \omega_c \sim k_B T$ ) but leave the room-temperature properties almost unscathed. The calculations of mobility as a function of sheet density, and the extracted value for the Hall scattering factor at room temperature, should therefore not be invalidated. Further investigations into the long-term stability of micron and sub-micron scale devices with a high level of grown-in strain is certainly necessary.

The resistivity as a function of temperature has been analyzed, and shown not to agree with existing theories of weak-localization and screening. A new functional form has been proposed as an empirical fit, but now needs theoretical justification.

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